Developing Written Calculations

IN MATHS

65 + 47 =
60 + 40 + 5 + 7 =
100 + 12 = 112

Developing Written Calculations

The development of recording mathematical calculations should take place alongside and compliment the use of mental strategies.

As calculations become more complex, written methods become more important. There are many different phrases associated with written methods such as 'pencil and paper calculations', 'formal written method', 'written algorithms', 'personal jottings'.

A written method can be thought of as a structured annotation of a calculation, distinct from an informal or personal jotting which nobody needs to see. A jotting can eventually be discarded whereas a genuine written method has lasting value.

A useful written method is one that helps children to carry out a calculation and can be understood by others.

It should be a long-term aim of the primary school to provide pupils with the skills to be able to choose an efficient method – mental, written or calculator – that is appropriate for a given task.

It should also be the aim to ensure that all pupils are equipped with a standard written method for each of the four operations by the time they reach the end of Key Stage 2.

It should also be remembered that there is strong evidence from research that the introduction of formal written methods too early can undermine children's fluency with number.

Why do we need written methods?

There are a number of reasons why written methods may be useful. They can:

- Represent work that has been done practically
- Support mental calculations often in the form of jottings
- Record and explain mental calculations
- Help in observing patterns
- Help keep track of steps in longer tasks
- Develop mental imagery
- Work out calculations that are too difficult to do mentally
- Develop efficiency in calculation

Written recording is needed to help us keep track of where we are in our calculation and to help explain our thinking.

When do children need to start recording?

Children should be encouraged to see mathematics as a written as well as a spoken language. Teachers need to support and guide children through the following important stages:

- Developing the use of pictures and a mixture of words and symbols to represent numerical activities
- Use of standard symbols and conventions, such as numerals 0 to 9, the equals sign and the operations signs to record mental calculations
- Use of jottings to aid a mental strategy
- Use of expanded forms of recording as a step towards standard paper and pencil methods
- Use of compact forms of recording
- Use of a calculator

It is important to encourage children to look first at the problem or calculation and then get them to decide which is the best method to choose – pictures, mental calculation with or without jottings, structured recording or calculator.

HORIZONTAL RECORDING

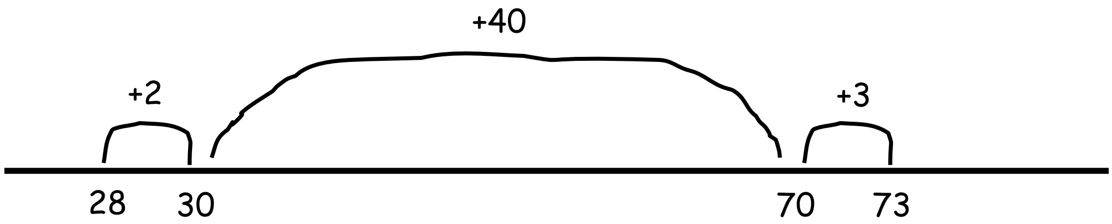
The main emphasis for recording of calculations at Key Stage 1 should be in a horizontal format. This will help children to make a record of the mental strategies they are using. The strategy children will use will depend on the numbers involved and the individual child. There are seven main strategies for mental calculation (see appendix 1)

Partitioning	47 + 66 = 40 + 60 + 7 + 6 = 100 + 13 = 113	or	47 + 66 = 47 + 60 + 6 = 107 + 6 = 113
	73 – 35 = 73 – 30 - 5 = 43 – 5 = 38		
Rounding and Adjusting	73 + 28 = 73 + 30 – 2 = 103 – 2 = 101		92 – 39 = 92 – 40 + 1 = 52 + 1 = 53
Re-ordering	7 + 9 + 13 + 8 + 4 13 + 7 = 20 9 + 8 = 17 20 + 17 = 37 + 4 = 41	 (components of 10/20) (near doubles)	

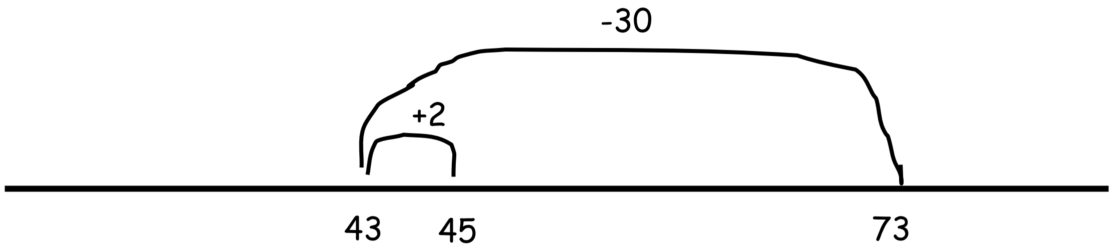
The empty number line is also a very valuable and flexible device for recording calculations:

For example, the answer to **73 - 28** could be calculated using either of the strategies below and represented on the empty number line as shown:

(a) by counting on to find the difference (bridging to significant landmarks or multiples of 10)



(b) by subtraction using rounding and adjusting



In order to calculate mentally and record their thinking as a horizontal calculation children need to have quick recall of basic number facts. Relationships between numbers should be emphasised and the inverse nature of addition and subtraction should be explored as should the inverse nature of multiplication and division. Number fact families should be built from number trios.

For example

13 4 9

4 + 9 = 13
9 + 4 = 13
13 – 4 = 9
13 – 9 = 4
13 = 4 + 9
13 = 9 + 4
9 = 13 – 4
4 = 13 - 9

7 28 4

4 x 7 = 28
7 x 4 = 28
28 ÷ 4 = 7
28 ÷ 7 = 4
28 = 4 x 7
28 = 7 x 4
4 = 28 ÷ 7
7 = 28 ÷ 4

In developing the mental strategies for calculation it is important to identify the skills required and to consider if there are any smaller steps or 'mini-skills' that need to be established first. It is important to provide time for children to acquire these contributing skills.

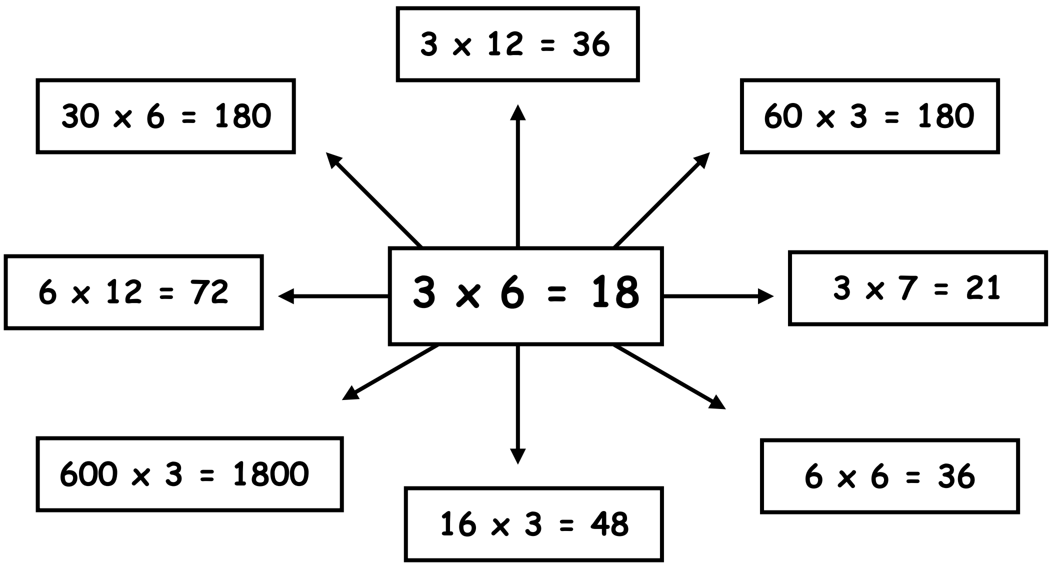
For example, if you are encouraging pupils to mentally calculate 73 x 6 and record it horizontally they will need the following contributing skills or knowledge:

- quick recall of number facts to 10x10
- knowledge of place value (73 = 70 + 3)
- knowledge of partitioning 73 x 6 = (70 x 6) + (3 x 6)
- ability to multiply a multiple of 10 by a single digit (70 x 6) derived from the basic number fact 7 x 6

Time should be spent considering number facts and using them to derive other number facts. For example,

- 4 + 3 = 7, 14 + 3 = 17, 24 + 3 = 27, etc.
- 8 – 2 = 6, 18 – 2 = 16, 28 – 2 = 26, etc.
- 9 + 5 = 14, 90 + 50 = 140, 900 + 500 = 1400, etc.
- 7 x 4 = 28, 70 x 4 = 280, 700 x 4 = 2800, 70 x 40 = 2800, 0.7 x 4 = 2.8, etc.

What other multiplication facts can you work out from 3 x 6 = 18? Give children time to talk about and explain why they were able to work out the new facts identified.



Readiness for Formal Written Methods

The National Numeracy Strategy guidance (1999) identifies a list of criteria for readiness for formal written methods of calculation.

An affirmative answer to the follow questions should indicate readiness for formal methods of calculation in **addition and subtraction** :

- Do the children know addition and subtraction facts to 20?
- Do they understand place value and can they partition numbers into hundreds, tens and ones?
- Do they use and apply the commutative and associative laws of addition?
- Can they add at least three single-digit numbers mentally?
- Can they add and subtract any pair of two-digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?

Corresponding criteria to indicate readiness to learn formal written methods for **multiplication and division** are:

- Do the children know the 2, 3, 4, 5 and 10 times tables and the corresponding division facts?
- Do the children know the result of multiplying by 0 or 1?
    - Do they understand place value?
    - Do they understand zero as a place holder?
    - Can they multiply two and three-digit numbers mentally by 10 and 100?
- Can they use their knowledge of all the multiplication tables to approximate products and quotients using powers of 10?
- Do they use the commutative and associative laws for multiplication, and the distributive law of multiplication over addition or subtraction?
- Can they double and halve two-digit numbers mentally?
- Can they use multiplication facts they know to derive mentally other multiplication facts that they do not know?
- Can they explain their mental strategies orally and record them using informal jottings?

Appendices 2 and 3 give a range of questions based on the criteria above which children should be able to answer before moving on to formal written methods.

The teacher must make formative assessment judgements about children's grasp of mental calculation strategies before deciding whether to begin teaching formal written calculations.

RECORDING IN EXPANDED FORM

Using expanded forms of recording as a bridge to formal recording will naturally build on mental strategies.

Example 1 : Alison has £378. Her aunt gives her another £46. How much money does Alison have now?

The expanded recording for this calculation could be written as follows:

$$\begin{array}{rrrrrr} & 300 & + & 70 & + & 8 \\ + & & & 40 & + & 6 \\ \hline & 300 & + & 110 & + & 14 \\ \hline \end{array} = \text{£}424$$

Example 2 : There are 622 seats in a concert hall. There were 374 people at a concert. How many empty seats were there?

The expanded recording for this calculation could be written as follows:

$$\begin{array}{rrrrrr} & 600 & + & 20 & + & 2 \\ - & 300 & + & 70 & + & 4 \\ \hline & 300 & - & 50 & - & 2 \\ \hline \end{array} = 248 \text{ empty seats}$$

The subtraction example above does not imply a sophisticated understanding of negative numbers. The recording of '-50' simply represents 'I'm supposed to take away 70 from 20, I can only take away 20 so I have 50 more to take away' Similarly the recording of '-2' represents 'I'm supposed to take 4 away from 2, I can only take 2 away so I have 2 more to take away'.

Example 3a : There are 18 biscuits in a packet. How many biscuits would there be in 7 packets?

In this multiplication calculation the 'grid' or 'area' method can be used to link with mental calculation.

	10	8
7	70	56

70 + 56 = 126

Example 3b : There are 18 biscuits in a packet. How many biscuits would there be in 63 packets?

The 'grid' or 'area' method can easily be extended to long multiplication.

	10	**8**
60	**600**	**480**
3	**30**	**24**

600 + 480 + 30 + 24 = 1134

Example 4 : 96 apples are to be sold in packets of 4. How many packets will there be?

It is generally helpful to identify division with repeated subtraction and use the knowledge of multiplication facts to help. This can be done by 'chunking' 96 into numbers that are readily associated with 4.

$$\begin{array}{rl} 96 & \\ -\ 40 & \text{(10 packets)} \\ \hline 56 & \\ -\ 40 & \text{(10 packets)} \\ \hline 16 & \\ -\ 16 & \text{(4 packets)} \\ \hline 0 & \text{(24 packets)} \end{array}$$

VERTICAL RECORDING

If the criteria for readiness for formal written recording includes the ability to add and subtract two 2-digit numbers mentally it would suggest that formal written recording only becomes necessary when children are required to manipulate larger numbers (3-digits and more) which cannot easily be calculated mentally.

When developing formal vertical recording methods the use of structured apparatus should be used to develop the concept of exchange and the recording should mirror what is being done practically.

By the end of Key Stage 2 pupils should understand and use a concise formal method of recording calculations and most importantly know when the use of such a method is appropriate. An example of a concise form of recording for each operation is shown below:

ADDITION

$$\begin{array}{r} 7\,2\,9\,4 \\ +\ \ 3\,5_{1}8_{1}8 \\ \hline 1\,0\,8\,8\,2 \\ \hline \end{array}$$

SUBTRACTION

$$\begin{array}{r} \,{\scriptstyle 6}\ {\scriptstyle 13} \\ 9\,\cancel{7}\,\cancel{3} \\ -\ 6\,4\,8 \\ \hline 3\,2\,5 \\ \hline \end{array}$$

MULTIPLICATION

$$\begin{array}{r} 1\,8\,7 \\ \times_{3}\ {}_{2}\,4 \\ \hline 7\,4\,8 \\ \hline \end{array}$$

DIVISION

$$\begin{array}{r|l} & 1\,2\,3 \ \text{rem } 1 \\ \cline{2-2} 6 & 7\,{}^{1}3\,{}^{1}9 \end{array}$$

From school to school, the recording may vary slightly; for example, where the exchange is recorded in addition and multiplication, the quotient recorded below the dividend in division and the way in which the exchange is recorded in subtraction. Whatever approach is used, it is important that it is discussed and agreed throughout the school.

Long multiplication may be initially introduced through the 'area' or 'grid' method. The use of factors can also be used for certain calculations, for example;

$23 \times 42 = 23 \times 6 \times 7$

$$\begin{array}{r} 2\,3 \\ \times\ \ {}_{1}6 \\ \hline 1\,3\,8 \\ \times_{2}\ {}_{5}7 \\ \hline 9\,6\,6 \end{array}$$

Leading to a concise method of recording, the use of two separately recorded multiplication calculations can form a bridge, for example;

41×93 can be calculated as **$(41 \times 90) + (41 \times 3)$**

$$\begin{array}{r} 4\,1 \\ \times\ \ 9\,0 \\ \hline 3\,6\,9\,0 \\ \hline \end{array} \qquad \begin{array}{r} 4\,1 \\ \times\ \ \ 3 \\ \hline 1\,2\,3 \\ \hline \end{array} \qquad \begin{array}{r} 3\,6\,9\,0 \\ +\ \ 1_{1}2\,3 \\ \hline 3\,8\,1\,3 \\ \hline \end{array}$$

This can later be recorded more concisely:

$$\begin{array}{rl} 4\,1 & \\ \times\ \ 9\,3 & \\ \hline 3\,6\,9\,0 & (41 \times 90) \\ +\ \ 1_{1}2\,3 & (41 \times 3) \\ \hline 3\,8\,1\,3 & \\ \hline \end{array}$$

The following are examples of calculations in which a vertical layout is simply not helpful because they are best approached as a mental calculation

$$\begin{array}{r} 4000 \\ -\quad 70 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} £3.99 \\ \times \quad 7 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} £2.00 \\ +£1.98 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 16 \\ -\ 9 \\ \hline \\ \hline \end{array}$$

When approaching any calculation it is important to consider which method of calculation is most appropriate and use this method efficiently.

A useful question to ask is :

What could I change about this calculation to make it easier?

Exploring patterns of equivalent calculations can help children to adjust calculations to make them easier, for example,

Consider **136 + 48** and look at the patterns of equivalent calculations which can be made

136 + 48
137 + 47
138 + 46
139 + 45
140 + 44
141 + 43
142 + 42
143 + 41
144 + 40

Notice how the calculations maintain their equivalence. (adding 1 to one of the numbers and subtracting one from the partner number) Encourage children to look at the list and discuss which calculations are easier and why.

This can also be extended to subtraction calculations

Consider **275 – 137** and look at the patterns of equivalent calculations which can be made

275 – 137
274 – 136
273 – 135
272 – 134
271 – 133
270 – 132
269 – 131
268 – 130

Notice how the calculations maintain their equivalence. (subtracting the same amount from each number keeps the difference the same)
Encourage children to look at the list and discuss which calculations are easier and why.

This can lead to changing subtraction calculations with 'zeros' to a more manageable calculation, for example, 1002 – 437 could become 999 – 434

Another useful strategy to promote when adding strings of several numbers is to break the calculation into smaller strings. This minimises calculation errors when adding long strings of numbers

For example : **The number of bottles of milk sold in a supermarket over a 7-day period was as follows; 724, 562, 673, 824, 501, 588, 769 Calculate the mean number of bottles sold each day during the week.**

This represents 724 + 501 calculated mentally

$$\begin{array}{r} 1225 \\ +\ 562 \\ \hline 1787 \\ \hline \end{array} \qquad \begin{array}{r} 673 \\ +\ 824 \\ \hline 1497 \\ \hline \end{array} \qquad \begin{array}{r} 588 \\ +\ 7_{1}6_{1}9 \\ \hline 1357 \\ \hline \end{array} \qquad \begin{array}{r} 1787 \\ 1497 \\ +\ 1_{1}3_{2}5_{2}7 \\ \hline 4641 \\ \hline \end{array} \qquad \begin{array}{r|l} & 663 \\ \hline 7 & 4^{4}6^{4}4^{2}1 \end{array}$$

Ask pupils to consider different sets of calculations suited to the range of numbers and operations with which they have been working. Ask them to sort the calculations into two groups :

(a) calculations which can be performed mentally
(b) calculations which require a written method

Discuss which calculations can be done mentally and which require a written calculation. Ask them to explain their reasons.

The following pages contain 6 sample sets of calculations but the calculations you use should be chosen appropriately for your pupils.

Set 1

627 + 300	437 + 687	584 + 120
966 + 298	947 + 579	190 + 270
475 + 348	661 + 276	449 + 449

Set 2

678 - 131	900 - 72	913 - 588
765 - 110	507 - 291	841 - 199
723 - 719	398 - 150	225 - 175

Set 3

482 - 18	517 + 140	839 - 684
337 + 502	193 + 607	258 - 249
860 - 385	111 + 428	724 - 558

Set 4

3000 - 2	98 x 3	578 + 1938
507 x 4	3694 x 7	9000 - 62
3826 + 4997	324 x 5	8163 - 2875

Set 5

584 x 3	180 ÷ 6	1871 ÷ 7
9157 + 2387	260 x 3	5006 - 2518
4030 ÷ 2	1684 - 201	1807 - 1406

Set 6

£3.78 x 6	50% of £18.70	6.73 ÷ 9
98.2 – 14.37	$\frac{2}{5}$ of £5.75	23.81 + 4.9
17.3 + 4.6	1.8 x 3	6.29 + 4 + 3.9

Mental and Written Calculations

> **"A secure foundation of mental and oral work is essential if children are to develop their mathematical skills."**
>
> *Final Report of the Numeracy Task Force 1998*

Moving to written procedures too fast can mean:

Children adding two or three numbers cease to 'say' the numbers to themselves, but just add a column	56 + 29 = or 56 + 29 ___ ___	Instead of '56 add 29' children say '9 and 6'. They have no idea what numbers they are adding nor what a likely answer looks like.
Children using a written procedure miss the obvious.	2000 - 102 ___ ___	This is simply done by counting back, **not** decomposition.
Children relying on written procedures often look for a calculator if stuck, not a strategy.	25 x 8 = or 25 x 8 ___ ___	Mentally, this asks for the knowledge that 25 x 4 is 100. In vertical form, it requires a specific procedure.

<u>**Differences between Mental and Written Calculation**</u>

Mental	Written
We may break the calculation into manageable parts, *e.g. we do 148 – 100 + 1* *instead of 148 - 99*	We never change the calculation to an equivalent one, *e.g. 148 – 99 is done as it is.*
We say the calculation to ourselves, and therefore are aware of what numbers are involved, *e.g. 2000 – 10 is not much less than 2000*	We don't say the numbers to ourselves, but start a procedure such as: $\begin{array}{r} 148 \\ -\ 99 \\ \hline \\ \hline \end{array}$ by saying, "8 take away 9, you can't, so make it 18 take away 9…"
We choose a strategy to fit the numbers, *e.g. 148 – 99 may be done differently from 84 – 77, although they are both subtractions*	We always use the same method.
We draw upon specific mathematical knowledge, an understanding of the number system, learned number facts and so on.	We draw upon a memory of a procedure, and possibly, though not necessarily, an understanding of how it works.

In Summary

- Teaching written procedures too early can prejudice children's chances of developing efficient ways of working with numbers 'in their heads'.

- Mental calculation requires a rethink of the approach to place value. For vertical addition, we stress that 54 is 5 tens and 4 units, but for mental addition it is more crucial to think of 54 as 50 and 4 more.

- When doing mental calculations we:

  - read or 'say' the calculation to ourselves first, and therefore look more carefully at the numbers involved;

  - choose a strategy to fit the numbers, so that 45 – 29 is done in a different way from 45 – 37, and differently again from 45 – 6;

  - often change a calculation to make it easier to understand and do, e.g. 142 – 99 becomes 143 – 100, or to multiply by 25, multiply by 100 then divide by 4;

  - draw upon known facts, different ways of counting and our knowledge of how the number system works.

The Role of Written Calculations

Summary

- A useful written method is one that helps children to carry out a calculation and can be understood by others.
- It is essential to develop mental calculation strategies systematically with written calculations being reserved for those calculations which cannot be done mentally.
- Children should be encouraged to decide which is the best method to choose in carrying out a calculation – pictures, mental calculation with or without jottings, structured recording or calculator.
- Children should not use formal recording until they are secure in their understanding otherwise they simply try to remember rules.
- By the end of Key Stage 2 pupils should work towards knowing and understanding a compact standard method for each numerical operation.
- Vertical recording should be delayed until children are secure in their knowledge of basic facts and the way they may be used to derive new ones.
- Practice of a formal written method should be set in a variety of contexts, little and often, over a period of time. Practice is not just a case of repetition of many similar examples.
- There are various criteria which can be used to establish readiness for formal methods of written calculations.
- Pupils must be given opportunities to explain their methods using appropriate mathematical language.

- It is important to inform parents of any changes in approaches to calculation. Parents need to be reassured that the development of mental strategies and the possible delay in teaching written methods will not result in a drop in standards.

Appendix 1

Seven Strategies for Mental Calculation

There are only seven main strategies for Mental Calculation as detailed in the Northern Ireland Strategy for Numeracy Teaching and Learning file (2000).

Counting on / Counting back	When adding or subtracting count on in ones, twos, tens, hundreds, halves, tenths, etc.
Rounding and Adjusting	When adding near doubles, use doubles and adjust; when calculating round to the nearest 10/100 and adjust; when multiplying use known multiplication facts and adjust
Partitioning	When calculating keep one number intact and partition the other or partition both numbers and recombine
Re-ordering	Re-arrange numbers to make the calculation easier; look for components of 10, doubles, near doubles, etc.
Inverse Operations	Use the relationships between addition, subtraction, multiplication and division
Using Factors	When calculating use doubling and halving; when multiplying or dividing use multiples of 10 as a factor of one of the numbers; when multiplying or dividing by 50 or 25 use multiplication or division by 100
Using Equivalence	When calculating, comparing or ordering, select the appropriate form of fractions, decimals or percentages (Key Stage 2)

Counting on / Counting back

When adding or subtracting count on in ones, twos, tens, hundreds, halves, tenths, etc.

Rounding and Adjusting

When adding near doubles, use doubles and adjust; when calculating round to the nearest 10/100 and adjust; when multiplying use known multiplication facts and adjust

Partitioning

When calculating keep one number intact and partition the other or partition both numbers and recombine

Re-ordering

Re-arrange numbers to make the calculation easier; look for components of 10, doubles, near doubles, etc.

Inverse Operations

Use the relationships between addition, subtraction, multiplication and division

Using Factors

When calculating use doubling and halving; when multiplying or dividing use multiples of 10 as a factor of one of the numbers; when multiplying or dividing by 50 or 25 use multiplication or division by 100

Using Equivalence

When calculating, comparing or ordering, select the appropriate form of fractions, decimals or percentages.

Appendix 2

Readiness for Addition and Subtraction

Baseline Test

Name : ____________________

____

Class : ________________

Section A : Addition and subtraction facts to 20

5 + 6 = ☐	7 + 8 = ☐	4 + 4 = ☐
9 + 2 = ☐	6 + 7 = ☐	4 + 13 = ☐
16 + 3 = ☐	3 + 12 = ☐	7 + 7 = ☐
9 – 4 = ☐	11 – 5 = ☐	18 – 9 = ☐
17 – 12 = ☐	13 – 6 = ☐	20 – 4 = ☐

Section B : Place Value and partitioning

Example : 64 = 60 + 4

24 = ☐	52 = ☐
69 = ☐	73 = ☐
28 = ☐	41 = ☐

Section C : Commutative Properties

$5 + 7 = 7 + \boxed{}$

$6 + 8 = \boxed{} + 6$

$3 + 11 = 11 + \boxed{}$

$\boxed{} + 11 = 11 + 2$

$9 + \boxed{} = 4 + 9$

$7 + 14 = \boxed{} + 7$

Section D : Associative Properties

$5 + 3 + 2 = 3 + 2 + \boxed{}$

$2 + \boxed{} + 6 = 9 + 6 + 2$

$\boxed{} + 11 + 1 = 1 + 4 + 11$

$9 + 5 + 8 = 5 + \boxed{} + 9$

Section E : Adding three single digit numbers

$6 + 4 + 8 = \boxed{}$

$6 + 9 + 2 = \boxed{}$

$2 + 4 + 7 = \boxed{}$

$8 + 6 + 6 = \boxed{}$

$4 + 9 + 1 = \boxed{}$

$5 + 7 + 5 = \boxed{}$

Section F : Adding two-digit numbers mentally using a strategy of their choice

46 + 24 = $\square$	37 + 37 = $\square$
75 + 15 = $\square$	28 + 19 = $\square$
54 + 27 = $\square$	34 + 35 = $\square$

Section G : Subtracting two-digit numbers mentally using a strategy of their choice

57 – 23 = $\square$	84 – 19 = $\square$
62 – 14 = $\square$	80 – 15 = $\square$
61 – 58 = $\square$	92 – 28 = $\square$

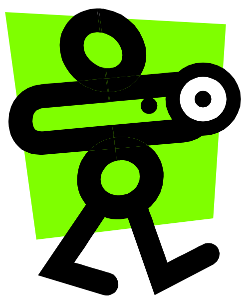
Appendix 3

Readiness for Multiplication and Division

Baseline Test

Name : ______________________

____



Class : ________________

Section A : 2, 3, 4, 5 and 10 times-tables and corresponding division facts

$3 \times 4 = \square$ $5 \times 7 = \square$ $10 \times 4 = \square$

$5 \times 5 = \square$ $9 \times 4 = \square$ $3 \times 9 = \square$

$4 \times 4 = \square$ $3 \times 10 = \square$ $5 \times 8 = \square$

$30 \div 5 = \square$ $24 \div 3 = \square$ $12 \div 4 = \square$

$60 \div 10 = \square$ $18 \div 2 = \square$ $20 \div 4 = \square$

Section B : Place Value, multiplying by 0 and 1

$7 \times 0 = \square$ $8 \times 1 = \square$ $0 \times 5 = \square$

$74 \times 10 = \square$ $88 \times 10 = \square$ $731 \times 10 = \square$

$54 \times 100 = \square$ $96 \times 100 = \square$

$\square \times 100 = 2100$

Section C : Use knowledge of multiplication tables to approximate products and quotients using powers of 10

$7 \times 100 = \square$ $\square \times 10 = 200$ $690 \div 10 = \square$

$100 \times 9 = \square$ $100 \times \square = 1100$ $140 \div \square = 14$

Section D : Commutative, associative and distributive laws

$3 \times 6 = 6 \times \square$ $7 \times 4 = \square \times 7$

$5 \times 4 \times 2 = 4 \times \square \times 5$ $\square \times 4 \times 3 = 4 \times 6 \times 3$

$7 \times 9 = 7 \times 5 + 7 \times \square$ $16 \times 6 = \square \times 6 + 6 \times 6$

Section E : Double and halve two-digit numbers mentally

Find half of the following numbers

$70 \longrightarrow \square$ $48 \longrightarrow \square$ $92 \longrightarrow \square$

Double the following numbers

$17 \longrightarrow \square$ $28 \longrightarrow \square$ $49 \longrightarrow \square$

Section F : Use known multiplication facts to derive other facts they do not know.

$4 \times 5 =$ $\boxed{}$ and $10 \times 5 =$ $\boxed{}$ so $14 \times 5 =$ $\boxed{}$

$10 \times 8 =$ $\boxed{}$ so $20 \times 8 =$ $\boxed{}$

$100 \times 7 =$ $\boxed{}$ so $300 \times 7 =$ $\boxed{}$